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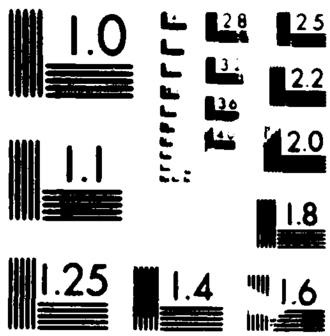

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Optical flow is the apparent (or perceived) motion of image brightness patterns arising from relative motion of objects and observer. Estimation of the optical flow requires the application of two kinds of constraint: the flow field smoothness constraint and the brightness constancy constraint. The brightness constancy constraint permits one to match image brightness values across images, but is very restrictive. We propose replacing this constraint with a more general constraint, which permits a linear transformation between image brightness values. The transformation parameters are allowed to vary smoothly, so that inexact

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Relaxing the Brightness Constancy Assumption in Computing Optical Flow

Michael A. Gennert and Shahriar Negahdaripour

Abstract: Optical flow is the apparent (or perceived) motion of image brightness patterns arising from relative motion of objects and observer. Estimation of the optical flow requires the application of two kinds of constraint: the flow field *smoothness constraint* and the *brightness constancy constraint.* The brightness constancy constraint permits one to match image brightness values across images, but is very restrictive. We propose replacing this constraint with a more general constraint, which permits a linear transformation between image brightness values. The transformation parameters are allowed to vary smoothly, so that inexact matching is allowed. We describe the implementation on a highly parallel computer, and present sample results.

Key Words: Optical Flow, Brightness Constancy Assumption, Passive Navigation, Smoothness Constraint.

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1 Introduction

Optical flow is the apparent (or perceived) motion of image brightness patterns arising from relative motion of objects and observer. Optical flow can give important information about motion of the observer (i.e. passive navigation), motion of objects in the scene, and the spatial arrangement of these objects. Additionally, discontinuities in the optical flow field can be used to segment the image into regions corresponding to different objects.

To be precise, an optical flow field is a two-dimensional vector field relating brightness patterns in an image at one instant of time to brightness patterns at the next instant of time. There does not exist a unique optical flow field for a given image sequence; rather there are infinitely many flow fields satisfying the image constraints. This illustrates one of the difficulties associated with determining optical flow, namely, identifying sufficient constraint to produce a unique optical flow field. The other difficulty in determining optical flow is more fundamental, and involves finding image elements to be placed into correspondence.

As in stereo, methods for computing optical flow can be classified according to whether detected features are used as primitive elements, or whether image brightness values (and gradients) are used directly. Feature-based approaches to optical flow use detected edges almost exclusively (Hildreth [1983], Davis et al [1983], Murray & Buxton [1984], Wohn [1984]). Although it is possible to use detected points to process visual motion, no one seems to have attempted to determine optical flow from isolated point displacements. Horn & Schunck [1981] were among the first to use image brightness directly to determine optical flow. They solved the problems mentioned above by identifying two constraints: the spatial *smoothness constraint* and the *brightness constancy constraint*. Other methods based on various smoothness assumptions have been proposed (Prager & Arbib [1983], Paquin & Dubois [1983], Yashida [1983], Anandan [1984]).

The smoothness constraint arises from the observation that most visual motion is the result of objects of finite size undergoing rigid motion or deformation. Neighboring object points have similar motions or velocities, and to the extent that they project to neighboring image points, neighboring image points will also have similar motions. Therefore, the optical flow field should be smooth almost everywhere. Exceptions occur at occluding boundaries, where neighboring image points are not generally the projections of neighboring object points.

The brightness constancy constraint rests on the assumption that the brightness of a small image patch remains approximately constant as the corresponding surface patch moves in the environment. This is a reasonable assumption when the lighting conditions are unchanged between successive images, object surfaces are non-specular, and there is only a small amount of motion between image frames. If these conditions are met, then the brightness constancy constraint will apply approximately at all image points.

It should be noted that the smoothness constraint depends on the scene structure, and is independent of illumination, surface reflectance characteristics, and the type and degree of motion involved. On the other hand, the brightness constancy constraint does not depend on scene structure (except for the influence of surface microstructure on reflectance), but instead depends on the degree and types of motion, and factors such as illumination and surface reflectance which affect image irradiance. Violations of the brightness constancy constraint, when they occur, affect image patches or even entire images. Therefore, it is important to find ways to relax this constraint.

Cornelius & Kanade [1983] propose a variation of the Horn & Schunck [1981] method. In their formulation, they allow gradual changes in the way an object appears in a sequence of images. An image point does not have to preserve the same brightness value as the object point that give rise to it moves in the environment, however, the variation is enforced to be smooth from one image point to the next.

In this paper, we propose a new formulation by relaxing the brightness constancy constraint. Our approach does not require exact brightness matching across image frames, but accepts even approximate matches. We achieve this by permitting a linear transformation of image brightness values between image frames, and constraining the allowed transformations. Our formulation, in special cases, reduces to that of Cornelius & Kanade [1983] or that of Horn & Schunck [1981].

2 Mathematics of Optical Flow

Let a coordinate system be aligned with the imaging system so that the z-axis points along the optical axis. The image plane can arbitrarily be chosen to lie at $z = 1$ so that image points are given by $\mathbf{r} = (x, y, 1)^T$. Let $E(\mathbf{r}, t)$ denote the brightness of image point $\mathbf{r}$ at time t. At a later time $t + \delta t$, the brightness pattern at $\mathbf{r}$ will have moved to a new location $\mathbf{r} + \delta\mathbf{r} = (x + \delta x, y + \delta y, 1)^T$. The optical flow is the velocity field arising from the perceived motion of image points $\mathbf{r}$. It is derived from the displacements of image points by taking the limit as $\delta t \to 0$,

$$\mathbf{r}_t = \frac{d\mathbf{r}}{dt} = (\frac{dx}{dt}, \frac{dy}{dt}, 0)^T = (u, v, 0)^T,$$

where u and v denote the components of optical flow $\mathbf{r}_t$.

The brightness constancy constraint of Horn & Schunck [1981] expresses the restriction that the brightness of an image patch remains approximately constant as the surface patch that gives rise to that image patch moves in the environment. Setting the total derivative of image brightness equal to zero, we can write

$$\frac{dE}{dt} = 0.$$

Applying the chain rule, we obtain

$$\frac{\partial E}{\partial t} + \frac{\partial E}{\partial x}\frac{dx}{dt} + \frac{\partial E}{\partial y}\frac{dy}{dt} = 0,$$

or

$$E_t + E_{\mathbf{r}} \cdot \mathbf{r}_t = 0,$$

where $E_{\mathbf{r}} = (\partial E/\partial x, \partial E/\partial y, 0)^T$. This equation is sometimes referred to as the *image brightness change constraint equation* derived under the constant brightness assumption. It has also been referred to as the *optical flow constraint equation.*

Assuming brightness constancy, we define optical flow as any 2D vector field $\mathbf{r}_t$, defined on the image plane, that satisfies the image brightness continuity equation.

2.1 Image Brightness Constraint

Nagel [1983a,1983b] suggests a formulation that incorporates second-order effects in order to obtain a better estimate of the optical flow around edges and corners. In his formulation, the brightness change constraint equation is written

$$E_t + E_{\mathbf{r}} \cdot \mathbf{r}_t + \frac{1}{2}\mathbf{r}_t^T E_{\mathbf{rr}} \mathbf{r}_t = 0,$$

where

$$E_{\mathbf{rr}} = \begin{pmatrix} \partial^2 E/\partial x^2 & \partial^2 E/\partial x \partial y & 0 \\ \partial^2 E/\partial x \partial y & \partial^2 E/\partial y^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Here, again, the constraint equation rests on the brightness constancy assumption.

The formulation proposed by Cornelius & Kanade [1983] allows gradual changes in the way an object appears in a sequence of images. In their formulation, the brightness change constraint is written

$$\frac{dE}{dt} = E_t + E_{\mathbf{r}} \cdot \mathbf{r}_t.$$

An image point does not have to preserve the same brightness value as the object point that gives rise to it moves in the environment. Hence, the rate of brightness change can be non-zero; that is,

$$\frac{dE}{dt} \neq 0.$$

More generally, we propose a formulation that allows a linear transformation between brightness values in consecutive images. We choose a linear transformation because it is one of the simplest non-trivial transformations. This is a less restrictive assumption than brightness constancy, and can be formulated as

$$E(\mathbf{r} + \delta\mathbf{r}, t + \delta t) = M(\mathbf{r}, t)\; E(\mathbf{r}, t) + C(\mathbf{r}, t),$$

where M is the multiplier and C is the offset functions in the linear transformation. This is our revised image brightness change constraint equation.

For small δt, we expect M to be close to 1, and C to be close to 0. Since we are dealing with incremental changes, we can let $M = 1 + \delta m$ and $C = \delta c$. In fact, m and c are the quantities of interest to us. Noting that $\delta m \to 0$ and $\delta c \to 0$ as $\delta t \to 0$, we can define time derivatives, m_t and c_t,

$$m_t = \lim_{\delta t \to 0} \frac{\delta m}{\delta t} \quad \text{and} \quad c_t = \lim_{\delta t \to 0} \frac{\delta c}{\delta t},$$

that we will use in our derivation.

Rewriting the brightness change constraint equation, we obtain

$$E(\mathbf{r} + \delta\mathbf{r}, t + \delta t) = [1 + \delta m(\mathbf{r}, t)]\, E(\mathbf{r}, t) + \delta c(\mathbf{r}, t).$$

The left hand side can be expanded as follows:

$$E(\mathbf{r} + \delta\mathbf{r}, t + \delta t) = E(\mathbf{r}, t) + \frac{\partial E}{\partial \mathbf{r}} \cdot \delta\mathbf{r} + \frac{\partial E}{\partial t}\delta t + O(\epsilon) = E_{\mathbf{r}} \cdot \delta\mathbf{r} + E_t\, \delta t + O(\epsilon).$$

Substituting this into the constraint equation and simplifying, we have

$$E_{\mathbf{r}} \cdot \delta\mathbf{r} + E_t\, \delta t - E\, \delta m - \delta c + O(\epsilon) = 0.$$

Finally, dividing through by δt and taking the limit as $\delta t \to 0$, we arrive at

$$E_t + E_{\mathbf{r}} \cdot \mathbf{r}_t - E\, m_t - c_t = 0.$$

This is our revised optical flow constraint equation. In the special case that $M = 1$ and $C \neq 0$ (and, hence, $m_t = 0$ but $c_t \neq 0$), this becomes similar to the constraint equation in the Cornelius & Kanade [1983] formulation:

$$c_t = E_t + E_{\mathbf{r}} \cdot \mathbf{r}_t.$$

Further, in the more restricted case that $M = 1$ and $C = 0$ (that is, $m_t = c_t = 0$), our constraint equation reduces to the one in the Horn & Schunck [1981] formulation:

$$E_t + E_{\mathbf{r}} \cdot \mathbf{r}_t = 0.$$

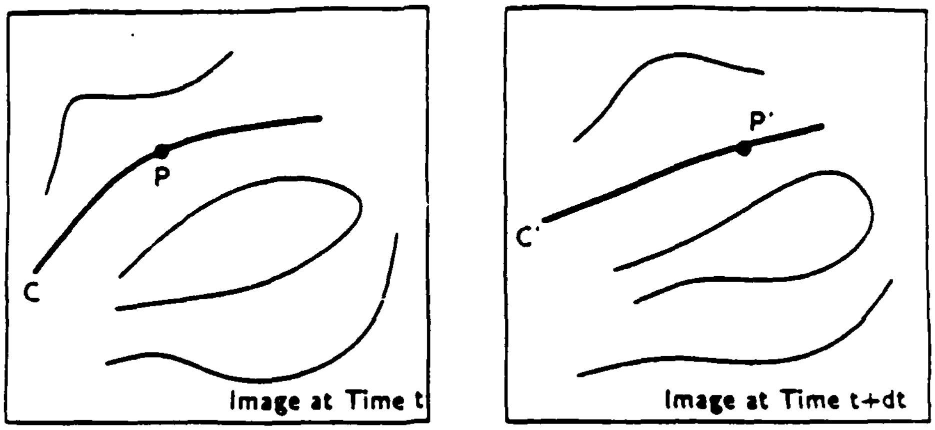

Figure 1. Corresponding iso-brightness contours in an image sequence.

3 The Aperture Problem

There are an infinite number of valid optical flows given an image sequence. To see this, we note that if $\mathbf{r}_t$ is an optical flow, so is

$$\mathbf{r}_t' = \mathbf{r}_t + f(x,y)\ (E_{\mathbf{r}} \times \hat{\mathbf{z}})$$

for any $f(x,y)$, where $\hat{\mathbf{z}}$ is a unit vector perpendicular to the image plane. First, note that $\mathbf{r}_t' \cdot \hat{\mathbf{z}} = \mathbf{r}_t \cdot \hat{\mathbf{z}} = 0$, as it should. Furthermore, we have

$$E_t + E_{\mathbf{r}} \cdot [\mathbf{r}_t + f\ (E_{\mathbf{r}} \times \hat{\mathbf{z}})] - E\, m_t - c_t = E_t + E_{\mathbf{r}} \cdot \mathbf{r}_t - E\, m_t - c_t = 0.$$

This can also be explained graphically. Consider the simple case where $M = 1$ and $C = 0$, so that the constraint equation reduces to that of Horn & Schunck formulation:

$$E_{\mathbf{r}} \cdot \mathbf{r}_t + E_t = 0.$$

Referring to figure 1, suppose that C' is a contour of constant brightness in the second image corresponding to contour C in the first image. It is not easy to decide which point P' on C' corresponds to a particular point P on C since the contour generally changes shape as the object moves in the environment (Horn [1986]). In fact, there are many possible ways to establish correspondence between points on contours C and C'. This ambiguity has been referred to as the *aperture problem*. In terms of the iso-brightness contours, any vector field that transforms contour C into contour C' is an acceptable optical flow.

In our extended formulation, the indeterminacy of the optical flow is even worse than this example suggests. Since m_t and c_t are unconstrained, the optical flow field can be completely arbitrary, with either or both of these transformation fields varying in such a way as to guarantee that the brightness constraint is obeyed. It is, therefore, necessary to select, out of the infinite number of possible optical flows, one which is consistent with the physical constraints of the problem. One may hope to obtain an optical flow field that approximates the apparent motion of brightness patterns in the image as judged by a human observer.

3.1 Smoothness Assumption

Discontinuities in depth (for example, at occluding boundaries) give rise to discontinuities in the optical flow field. Also, object motions may be different across occluding boundaries, which can give rise to discontinuities in the optical flow. Additionally, discontinuities can be expected in m_t and c_t, if illumination conditions or reflectance properties that depend on surface material change abruptly as the surface moves in the environment. In the absence of depth discontinuities or abrupt changes in illumination or surface reflectance properties, the optical flow and transformation fields are expected to be smooth. Based on these facts, we require that the optical flow, the multiplier, and the off-set fields should be consistent with our revised optical flow constraint equation, and should vary smoothly from one image point to the next.

Smoothness can be imposed by minimizing a functional that is a measure of departure from smoothness. Horn & Schunck [1981] proposed minimizing the integral of the square of the magnitude of the gradient of the optical flow. Hildreth [1983] investigated a similar formulation, but incorporated different measures of smoothness.

The gradient of the optical flow is

$$\nabla \mathbf{r}_t = \frac{\partial \mathbf{r}_t}{\partial \mathbf{r}} = \begin{pmatrix} \partial u/\partial x & \partial u/\partial y & 0 \\ \partial v/\partial x & \partial v/\partial y & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The measure of departure from smoothness that is to be minimized is written

$$e_s = \iint \|\nabla \mathbf{r}_t\|_2^2 \, dx \, dy.$$

Here, $\|\cdot\|_2^2$ denotes the Euclidean or Frobenius norm of a matrix, which is the sum of the square of all the elements of the matrix.

Similarly, smoothness deviations can be defined for the transformation fields

$$e_m = \iint \|\nabla m_t\|_2^2 \, dx \, dy \quad \text{and} \quad e_c = \iint \|\nabla c_t\|_2^2 \, dx \, dy.$$

4 Minimization

The image brightness constraint and the smoothness constraints can be combined by defining a single functional that weighs each contribution. Rather than enforcing the brightness change constraint exactly, we use a penalty term that measures the square of the error in the constraint equation over the whole image:

$$e_b = \iint (E_t + E_{\mathbf{r}} \cdot \mathbf{r}_t - E\, m_t - c_t)^2 \, dx \, dy.$$

To ensure that the optical flow and the transformation fields (approximately) satisfy the optical flow constraint equation, we want e_b to be small.

All together, the problem can be formulated as that of minimizing the functional

$$e = e_b + \lambda_s e_s + \lambda_m e_m + \lambda_c e_c,$$

where λ_s, λ_m, λ_c weigh the total error contributed by each term.

Using variational calculus, the Euler–Lagrange equations for this problem can be found. These equations form a set of necessary conditions that a solution to our minimization problem has to satisfy. Sufficiency is not guaranteed, in particular, it is possible for a particular proposed solution to obey the Euler–Lagrange equations yet not be a global minimum. This will occur at local minima, points of inflection, and local maxima. Note, however, that there is no global maximum, as the functional is not bounded from above. We will not address the question of sufficiency further in this paper.

The variational problem is solved by using the formula

$$\Psi_f - \frac{\partial}{\partial x}\Psi_{f_x} - \frac{\partial}{\partial y}\Psi_{f_y} = 0,$$

where Ψ is the integrand in the cost functional and f is each of u, v, m_t or c_t, in turn. Applying the above formula, we obtain

$$\begin{aligned}
\nabla^2 u &= \frac{E_x}{\lambda_s}(E_t + E_x u + E_y v - E\, m_t - c_t), \\
\nabla^2 v &= \frac{E_y}{\lambda_s}(E_t + E_x u + E_y v - E\, m_t - c_t), \\
\nabla^2 m_t &= \frac{-E}{\lambda_m}(E_t + E_x u + E_y v - E\, m_t - c_t), \\
\nabla^2 c_t &= \frac{-1}{\lambda_c}(E_t + E_x u + E_y v - E\, m_t - c_t).
\end{aligned}$$

For a well-posed problem, we need to specify the appropriate boundary conditions. In the absence of fixed boundary conditions (values of u, v, m_t, and c_t on the image boundaries), we need to specify *natural boundary conditions.* For our problem, the natural boundary condition is

$$(f_x, f_y)^T \cdot \hat{\mathbf{n}} = 0,$$

where $\hat{\mathbf{n}}$ is a unit vector perpendicular to the boundary. Again, f can be any one of u, v, m_t or c_t, in turn.

4.1 A Discrete Implementation

In the discrete domain, the Laplacian operator ∇^2 can be approximated as a center-surround operator

$$\nabla^2 f \approx \kappa(\overline{f} - f),$$

where $\overline{f}$ (the "surround") is an averaged or smoothed version of f. For example, we can use the following simple approximation:

$$\overline{f} = \frac{1}{4}(f_{i+1,j} + f_{i-1,j} + f_{i,j+1} + f_{i,j-1}).$$

The scale factor κ can be absorbed into the appropriate λ and, therefore, need not be considered further.

Substituting the approximation to the Laplacian in the Euler–Lagrange equation, derived earlier, we can write a single matrix equation

$$\mathbf{A}\,\mathbf{f} = \mathbf{g}(\overline{\mathbf{f}}),$$

where

$$\mathbf{f} = \begin{pmatrix} u \\ v \\ m_t \\ c_t \end{pmatrix}, \quad \mathbf{g}(\overline{\mathbf{f}}) = \begin{pmatrix} \lambda_s \overline{u} - E_x E_t \\ \lambda_s \overline{v} - E_y E_t \\ \lambda_m \overline{m_t} + E E_t \\ \lambda_c \overline{c_t} + E_t \end{pmatrix},$$

and

$$\mathbf{A} = \begin{pmatrix} E_x^2 + \lambda_s & E_x E_y & -E_x E & -E_x \\ E_x E_y & E_y^2 + \lambda_s & -E_y E & -E_y \\ -E_x E & -E_y E & E^2 + \lambda_m & E \\ -E_x & -E_y & E & 1 + \lambda_c \end{pmatrix}.$$

These equations have to be solved iteratively since the optical flow, at each image cell, depends on the average of the optical flow from the neighboring cells. The same is true for m_t and c_t.

Solving for the unknown fields, u, v, m_t, and c_t, we find that

$$\mathbf{f} = \mathbf{A}^{-1}\mathbf{g}(\overline{\mathbf{f}}).$$

where

$$\mathbf{A}^{-1} = \frac{1}{\alpha} \begin{pmatrix} \begin{smallmatrix} \lambda_c\lambda_m\lambda_s+\lambda_m\lambda_s+ \\ E^2\lambda_c\lambda_s+E_y^2\lambda_c\lambda_m \end{smallmatrix} & -E_xE_y\lambda_c\lambda_m & E_xE\lambda_c\lambda_s & E_x\lambda_m\lambda_s \\ -E_xE_y\lambda_c\lambda_m & \begin{smallmatrix} \lambda_c\lambda_m\lambda_s+\lambda_m\lambda_s+ \\ E^2\lambda_c\lambda_s+E_x^2\lambda_c\lambda_m \end{smallmatrix} & E_yE\lambda_c\lambda_s & E_y\lambda_m\lambda_s \\ E_xE\lambda_c\lambda_s & E_yE\lambda_c\lambda_s & \begin{smallmatrix} \lambda_s^2+ \\ (E_x^2+E_y^2+\lambda_s)\lambda_c\lambda_s \end{smallmatrix} & -E\lambda_s^2 \\ E_x\lambda_m\lambda_s & E_y\lambda_m\lambda_s & -E\lambda_s^2 & \begin{smallmatrix} E^2\lambda_s^2+ \\ (E_x^2+E_y^2+\lambda_s)\lambda_m\lambda_s \end{smallmatrix} \end{pmatrix},$$

and

$$\alpha = \lambda_m\lambda_s^2 + E^2\lambda_c\lambda_s^2 + (E_x^2 + E_y^2 + \lambda_s)\lambda_c\lambda_m\lambda_s.$$

This is a system of linear equations, which can be used to recover the optical flow u and v, and the transformation fields m_t and c_t.

The field equations can be solved iteratively, at every image cell, according to the equation

$$\mathbf{f}^{k+1} = \mathbf{A}^{-1}\,\mathbf{g}(\bar{\mathbf{f}}^k),$$

where k is the iteration number. The matrix $\mathbf{A}$ (or $\mathbf{A}^{-1}$) depends only on the observed data. It needs to be computed once, but it differs from point to point.

This formulation, in general, requires a lot of computation and is not really suitable for implementation on a serial machine. It can, however, be readily implemented on a highly parallel computer, such as the Connection Machine™. For a 128 by 128 image, the Connection Machine implementation runs approximately 1000 faster than a Symbolics 3640™ Lisp Machine implementation.

5 Examples

Example 1 – Multiplier Effect with No Offset: Figure 2 shows a pair of image frames from a synthetic motion sequence. Each image contains a background texture and a central texture; each texture is gaussian-smoothed uniform noise. Sharp texture boundaries between the background and the central circular object have been preserved in each image. The circular region undergoes rigid rotation between frames. In addition to the rotation of the central circular region, the second image has been multiplied by a factor which varies linearly from 0.75 in the lower left corner to 1.25 in the upper right corner, as shown in Figure 3a. An offset of zero was used and all λ parameters were set to 1. The computed multiplier field, shown in Figure 3b, varies in a range from 0.82 in the lower left corner to 1.22 in the upper right corner. Here the linear trend is clearly discernible. The computed optical flow after 100 iterations is shown in Figure 4 (the needles indicate flow direction and rate). The offset field for this example had negligible values, with absolute values not exceeding 0.002. This is to be expected, as the experiment was designed so that the offset field would not be needed.

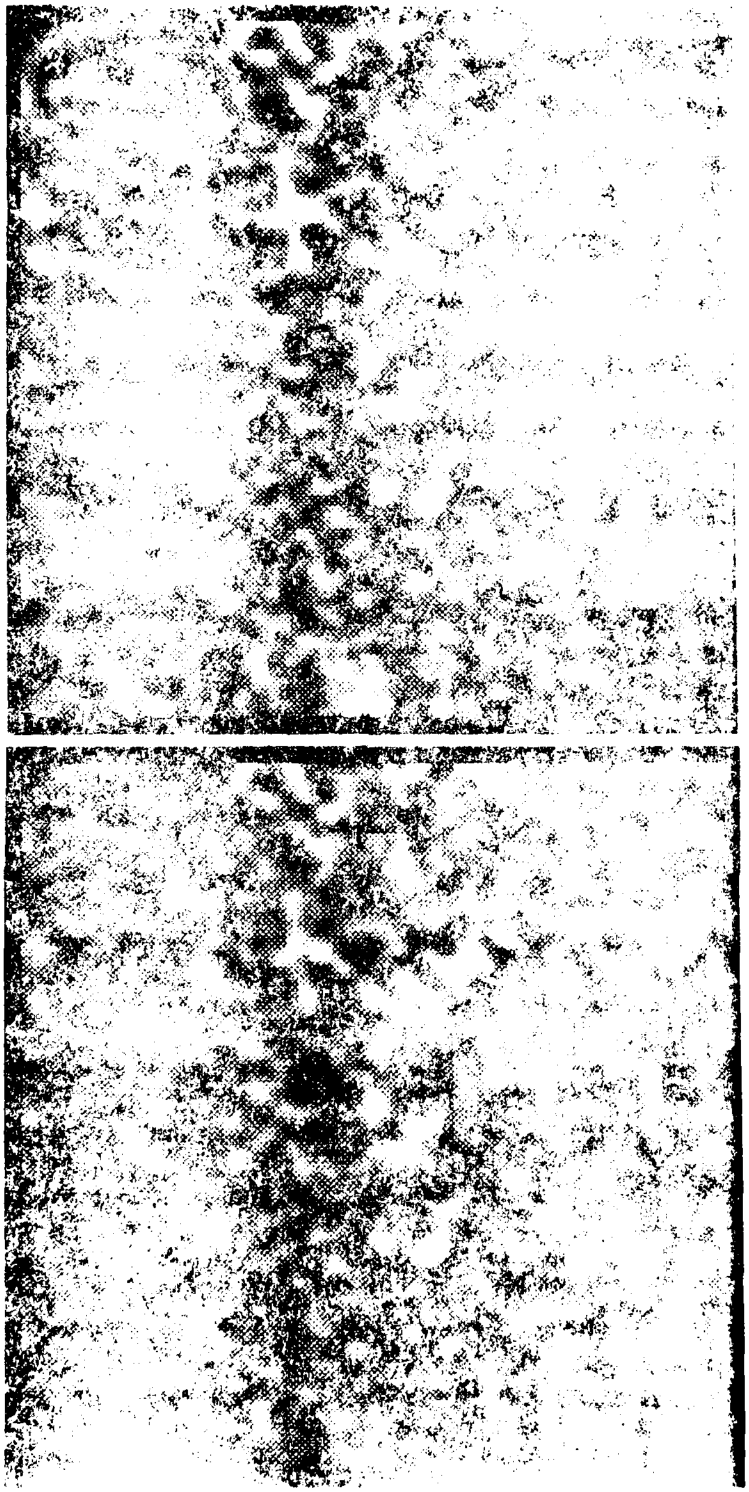

Figure 2. A pair of egion undergoes rigid rotation. In addition een scaled by a factor that varies linearly fi er.

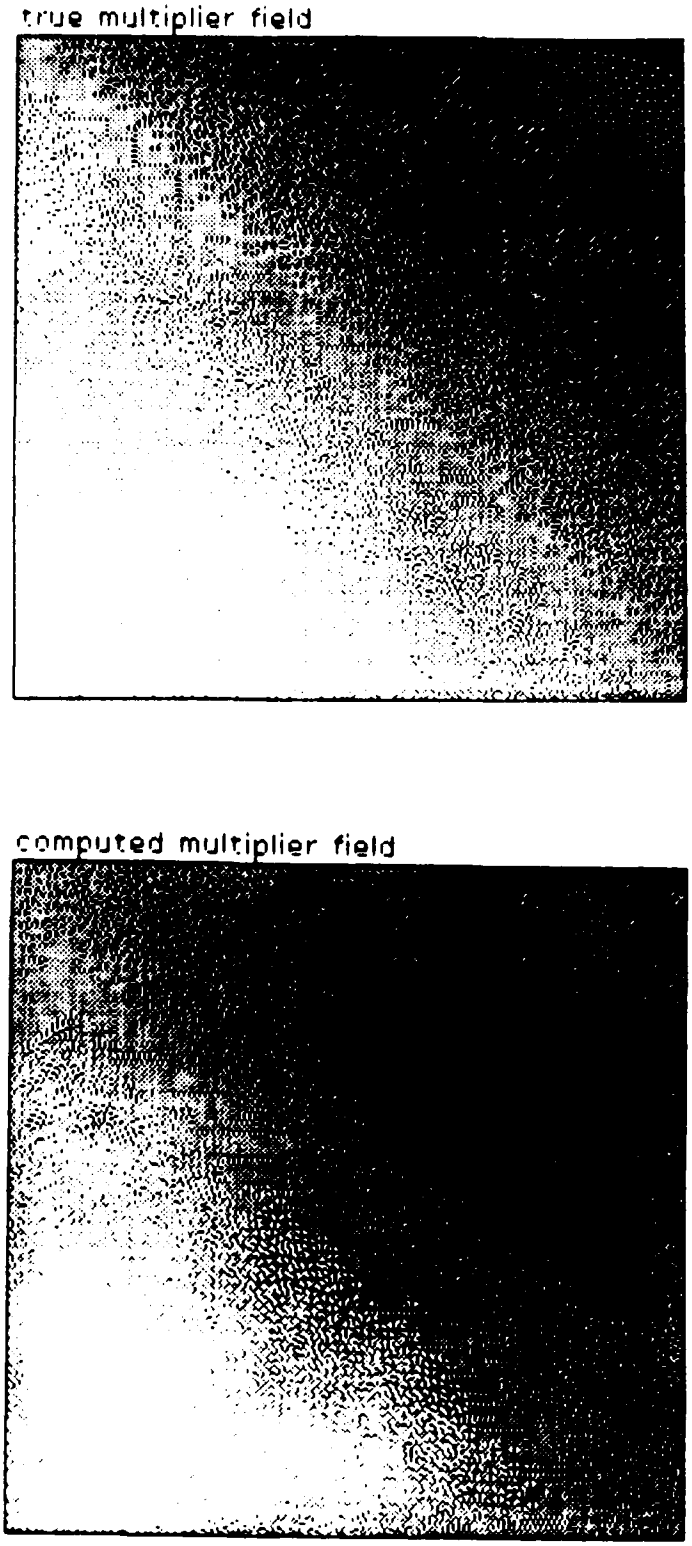

Figure 3. The true (a) and the computed multiplier fields used for the image sequence in **Figure 2**.

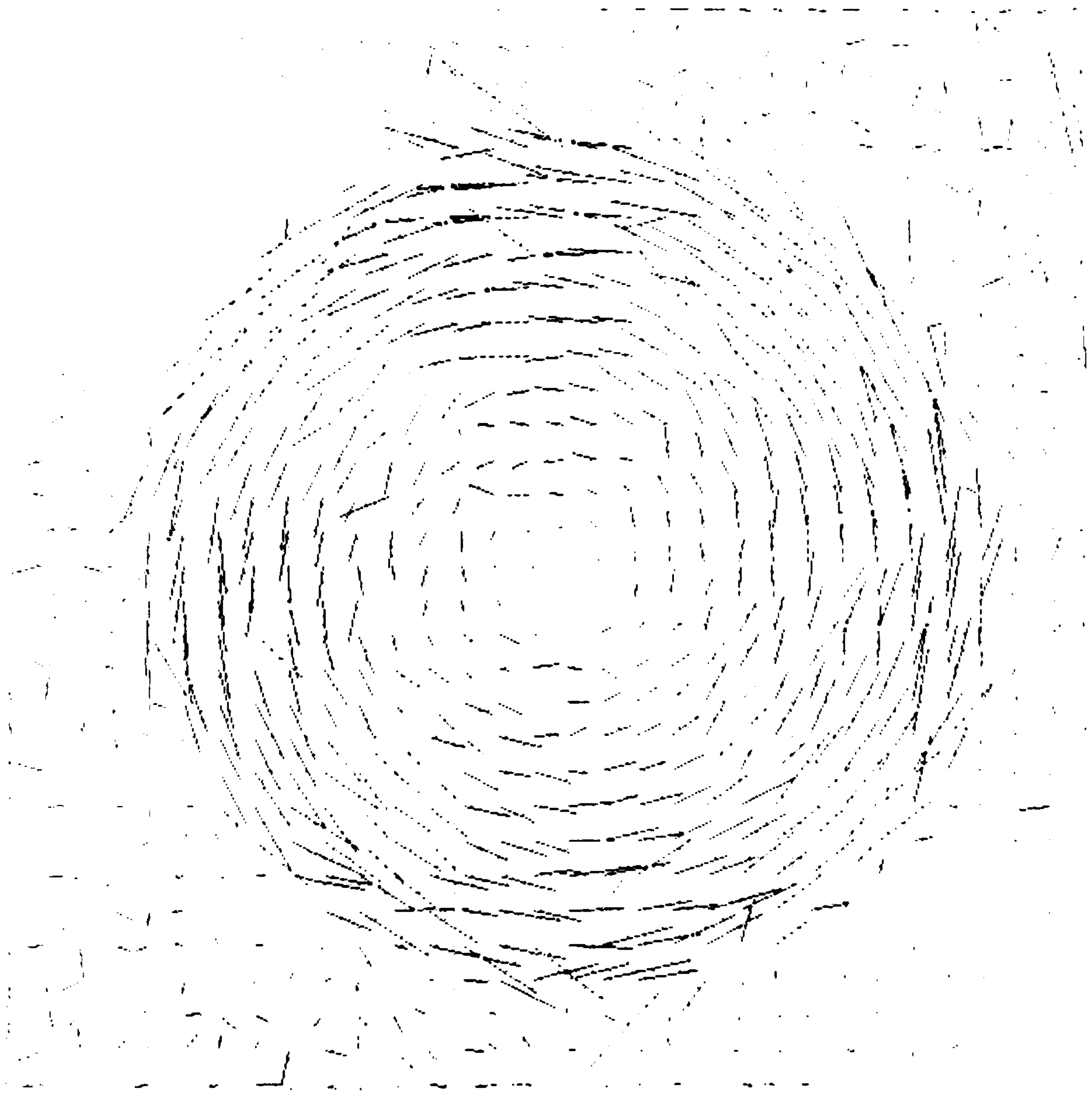

Figure 4. Example 1: The computed optical flow using the method described in this paper (all λ parameters were set equal to 1).

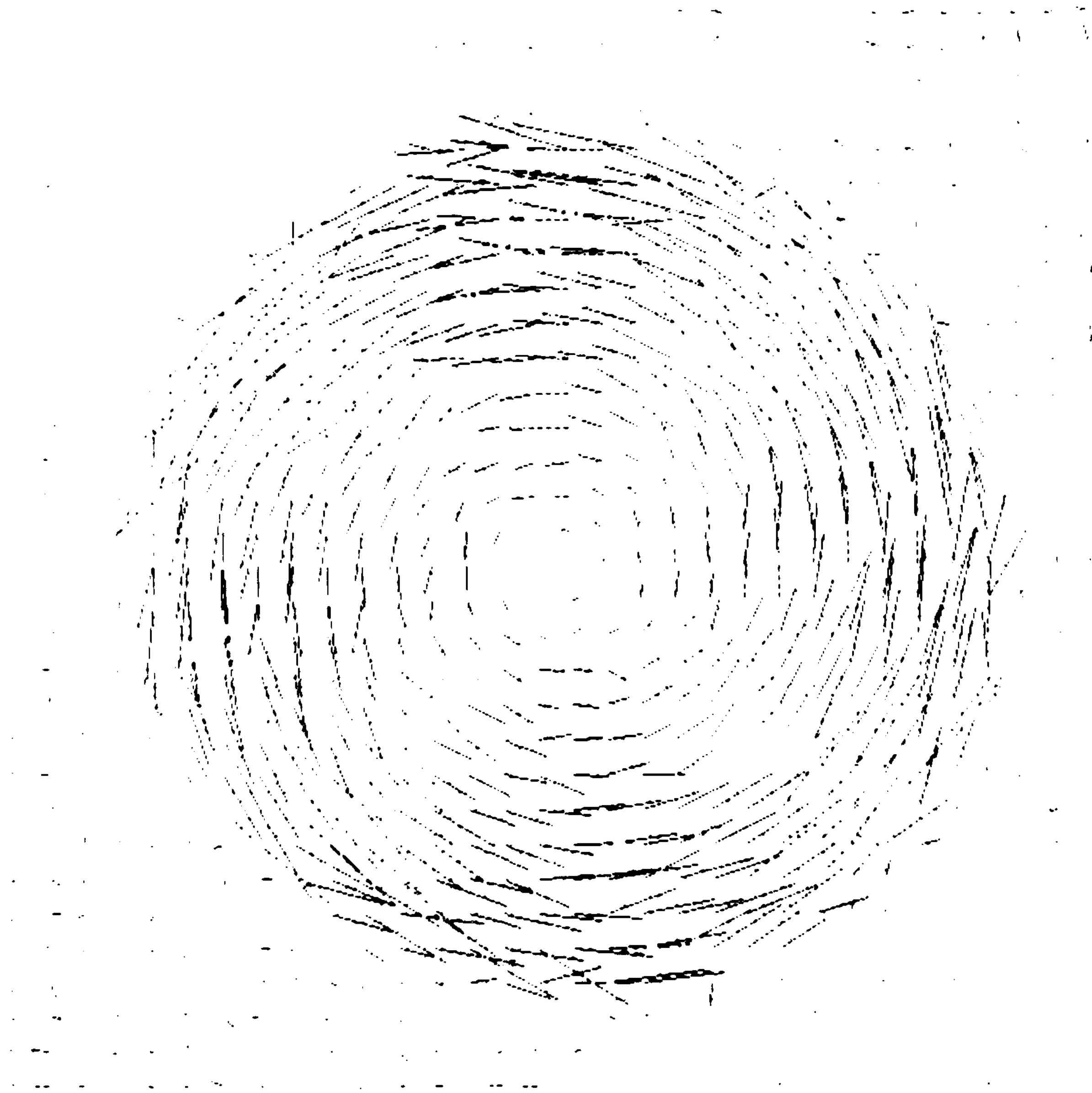

Figure 5. Example 1: The computed optical flow using the method described in this paper (λ_p = 0.1, but the remaining λ parameters were set equal to 1).

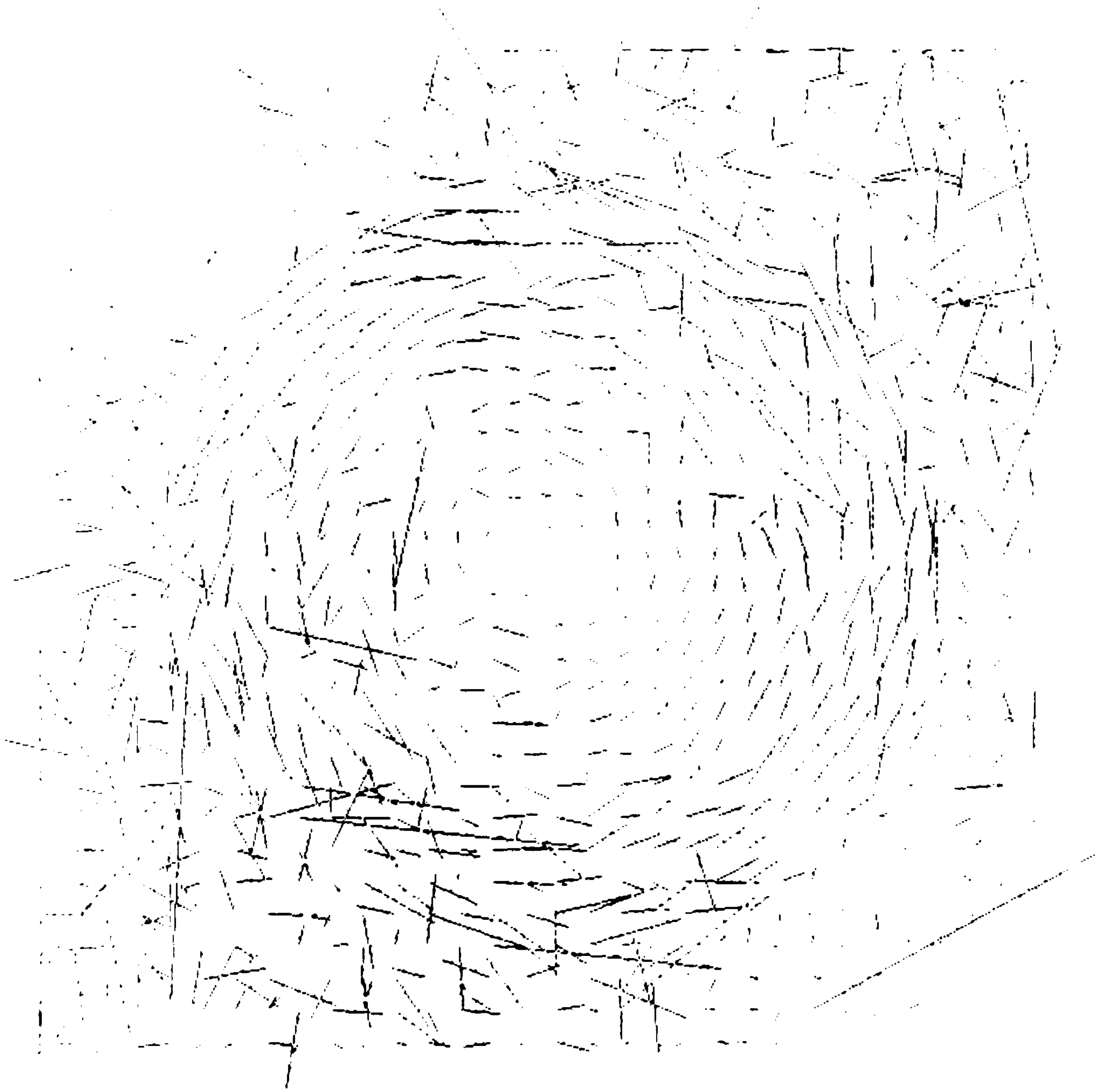

Figure 6. Example 1: The computed optical flow using the method of Horn & Schunck.

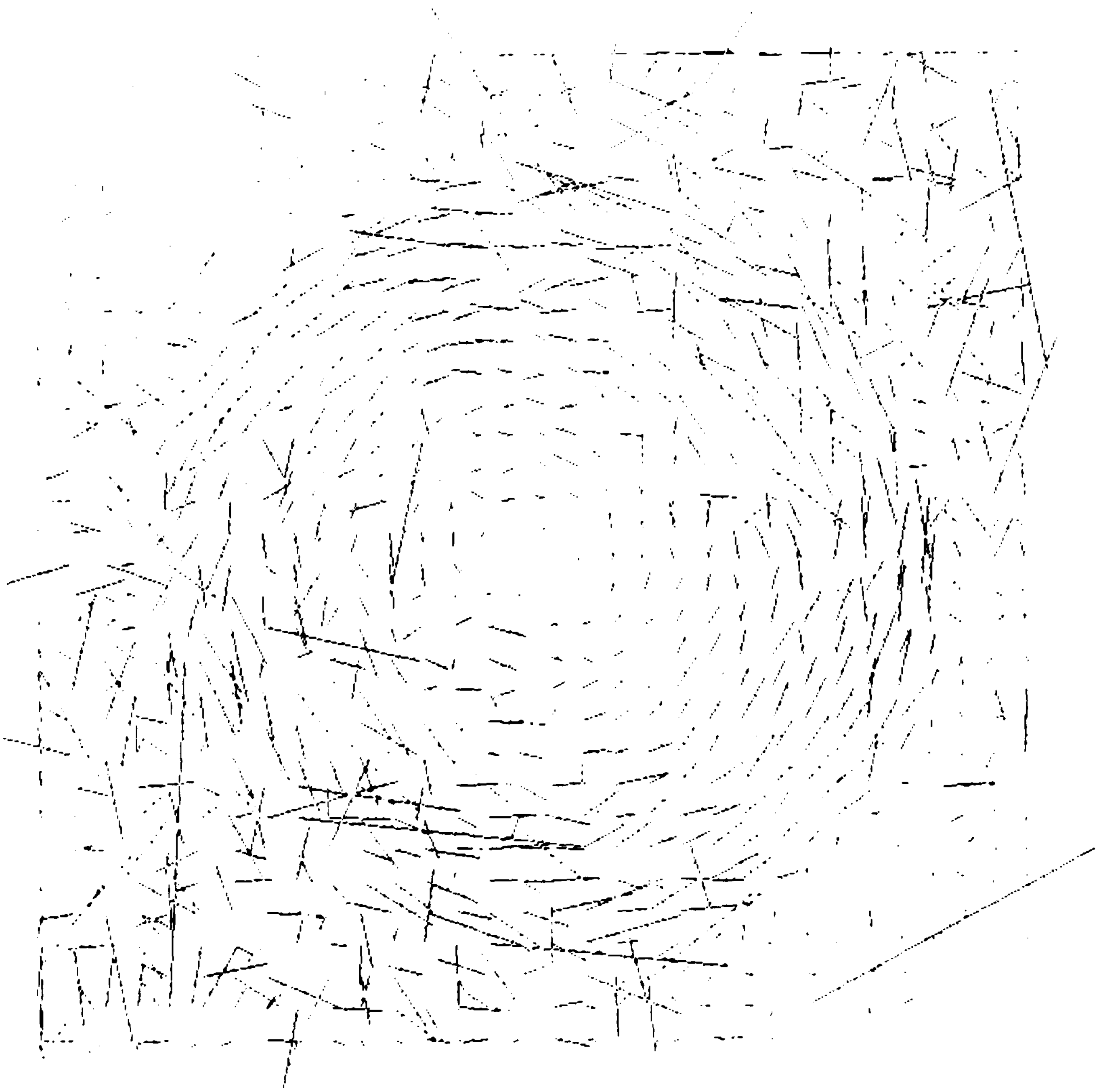

Figure 7. Example 1: The computed optical flow using the method of Cornelius & Kanade.

More accurate results were obtained for $\lambda_p = 0.1$ (all other λ parameters were set to 1) as shown in figures 5. The off-set field was negligible with absolute values not exceeding 0.0002, and the multiplier field varied in a range from 0.76 in the lower left corner to 1.26 in the upper right corner.

To check the improvement offered by this algorithm, the Horn & Schunck algorithm was also used on this image sequence. This is equivalent to using $\lambda_m = \lambda_c = \infty$ in our formulation. The results of the Horn & Schunck algorithm, Figure 6, were in agreement near the center of the image and the upper left and lower right corners, where the multiplier was approximately 1. The two algorithms did not agree, and the unmodified Horn &Schunck algorithm was clearly incorrect, at the lower left and upper right image corners where the multiplier had its greatest effect. This illustrates the inability of the Horn & Schunck algorithm to correctly handle images sequences where the brightness constancy constraint does not apply.

Figure 7 shows the solution obtained using the algorithm of Cornelius & Kanade. (This was done using $\lambda_m = \infty$ and $\lambda_c = 1$ in our formulation.) As expected, there is not much improvement over the solution from Horn & Schunck algorithm since their algorithm is designed to compensate for effects similar to an offset in an image sequence (however, the offset was set to zero for this example).

Example 2 – Multiplier and Offset Effects: Figure 8 shows the pair of images for this example. The motion is as in the previous case, the multiplier field varies linearly from 0.9 in the lower left corner to 1.1 in the upper right corner, and the offset is 5 (the grey-level values were increased by 5 units). The computed optical flow using our method is shown in Figure 9 (all λ parameters were set equal to 1). Figures 10 and 11 show the same using the methods of Horn & Schunck and Cornelius & Kanade, respectively. Again, these were obtained by setting $\lambda_m = \lambda_c = \infty$ (to simulate Horn & Schunck algorithm), and $\lambda_m = \infty$ and $\lambda_c = 1$ (to simulate Cornelius & Kanade algorithm) in our formulation. These results are reasonable where the multiplier and offset effects approximately cancel each other. They break down where the multiplier and offset have their greatest effects; that is, in the upper right corner of the image. We conclude that the Horn & Schunck algorithm does not correctly handle images sequences where the brightness constancy constraint is violated. Similarly, the algorithm of Cornelius & Kanade breaks down in regions of the image where multiplier effects are dominant.

6 Reducing the Computation

In many practical situations, the off-set c_t is near 0. Therefore, the algorithm can be

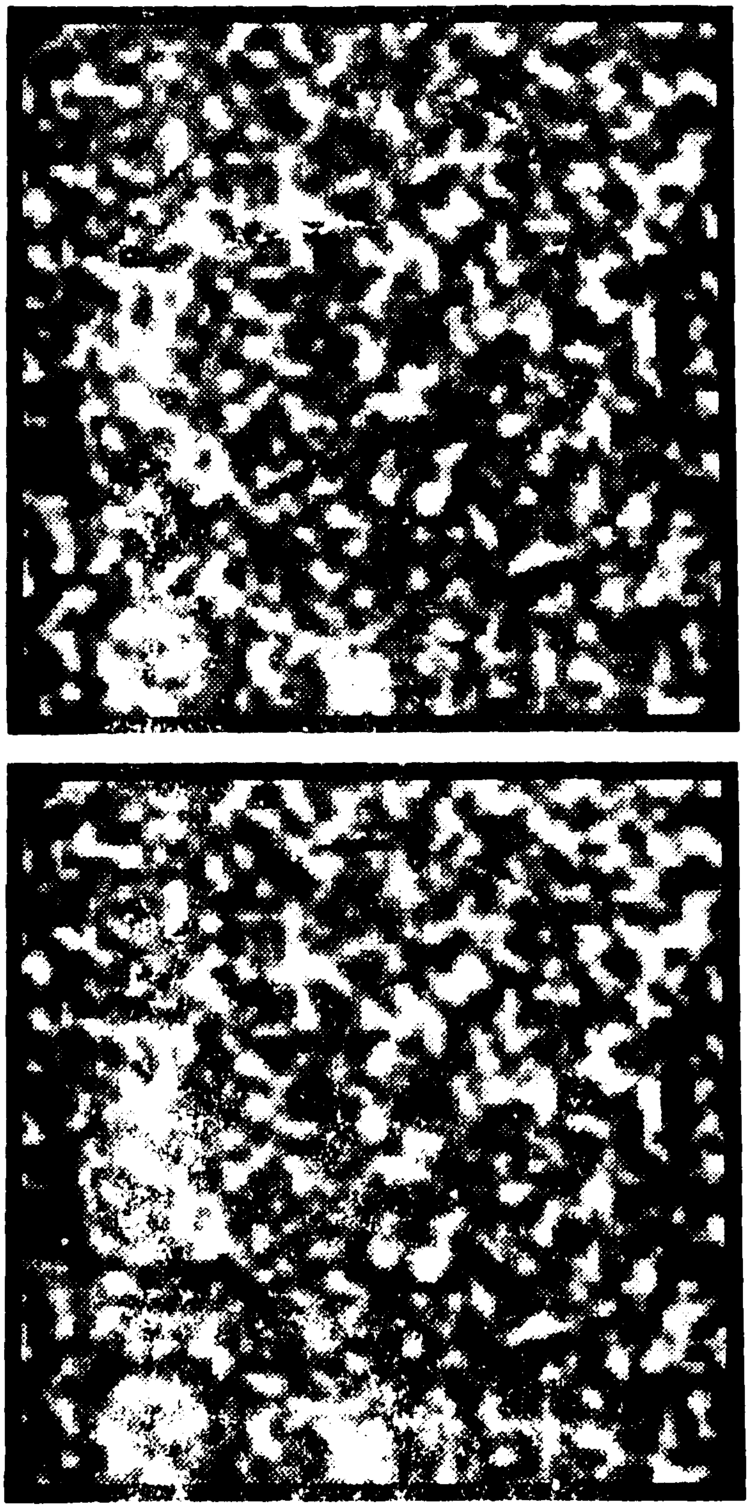

Figure 8. A pair of images from a synthetic motion sequence: the central circular region undergoes rigid rotation, the multiplier field varies linearly from [illegible] at the lower left corner to 1.1 the upper right corner, and the offset is 5 units.

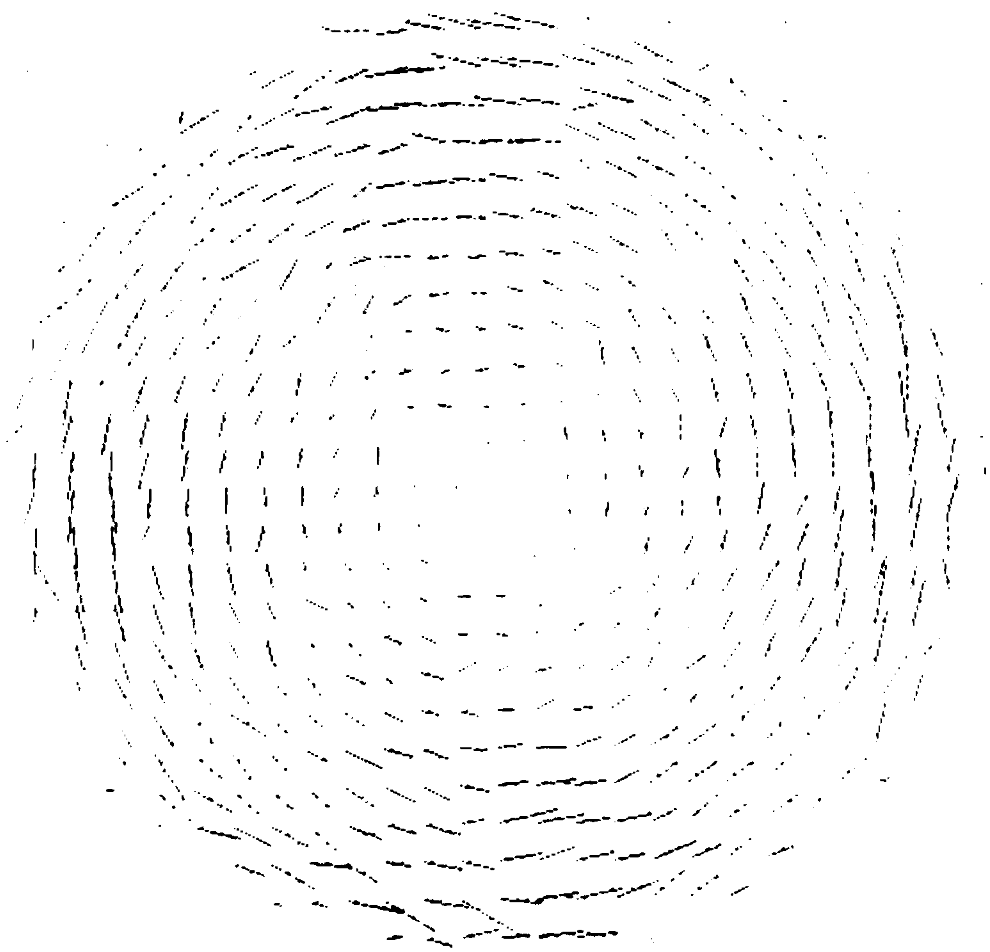

Figure 9. Example 2: The computed optical flow using the method described in this paper.

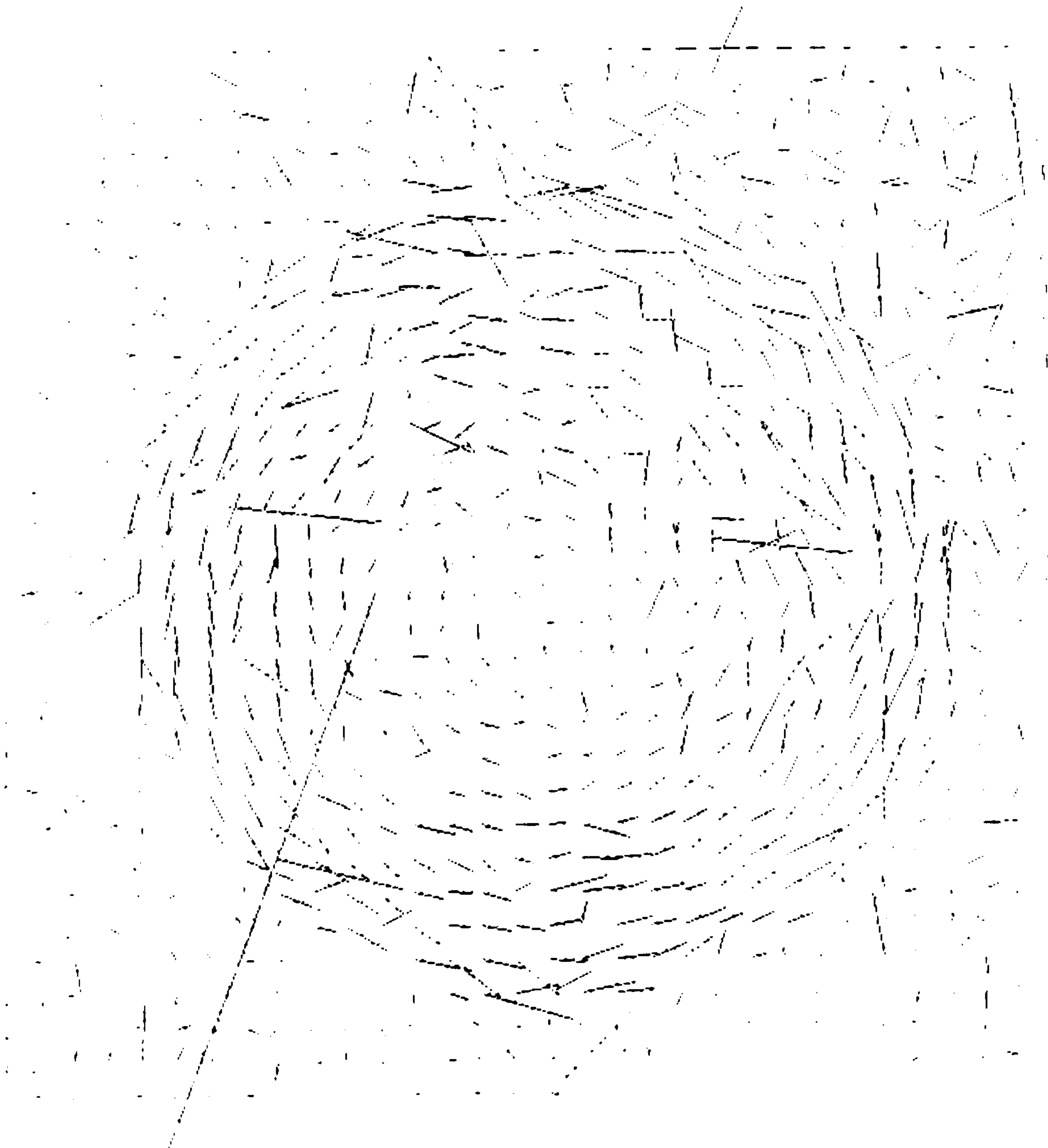

Figure 10. Example 2: The computed optical flow using the method of Horn & Schunck.

Figure 11. Example 2: The computed optical flow using the method of Cornelius & Kanade.

sped up by ignoring the offset term. We can instead use the updating equation

$$\mathbf{f}'^{k+1} = (\mathbf{A}')^{-1}\, \mathbf{g}'(\overline{\mathbf{f}}'^{k})$$

to compute the optical flow and the multiplier fields. Here, we have defined

$$\mathbf{f}' = \begin{pmatrix} u \\ v \\ m_t \end{pmatrix}, \quad \mathbf{g}'(\overline{\mathbf{f}}') = \begin{pmatrix} \lambda_s \overline{u} - E_x E_t \\ \lambda_s \overline{v} - E_y E_t \\ \lambda_m \overline{m_t} + E E_t \end{pmatrix},$$

and

$$(\mathbf{A}')^{-1} = \frac{1}{\alpha'} \begin{pmatrix} E_y^2 \lambda_m + E^2 \lambda_s + \lambda_m \lambda_s & -E_x E_y \lambda_m & E_x E \lambda_s \\ -E_x E_y \lambda_m & E_x^2 \lambda_m + E^2 \lambda_s + \lambda_m \lambda_s & E_y E \lambda_s \\ E_x E \lambda_s & E_y E \lambda_s & E_x^2 \lambda_s + E_y^2 \lambda_s + \lambda_s^2 \end{pmatrix},$$

where

$$\alpha' = \lambda_m \lambda_s^2 + E^2 \lambda_s^2 + (E_x^2 + E_y^2) \lambda_m \lambda_s,$$

If the offset term is not negligible, then the estimates obtained from the above vector equation may be used as initial conditions in the original updating equations.

7 Summary and Extensions

Much of the existing methods for computing the local optical flow depend on two kinds of constraint: the flow field *smoothness constraint* and the *brightness constancy constraint*. The brightness constancy constraint permits one to match image brightness values across images. This constraint is sometimes very restrictive.

We have proposed a new formulation by replacing the brightness constancy constraint with a more general constraint, which permits a linear transformation between image brightness values. The transformation parameters are allowed to vary slowly in space, so that inexact matching is allowed. We have formulated the problem of computing the optical flow as a minimization of a quadratic cost functional. Using variational methods, we have shown that the problem reduces to solving Laplacian equations for the two components of the optical flow field and the two transformation fields. We have described the implementation on a highly parallel computer, and presented sample results.

One of the disadvantages associated with the use of the smoothness constraint is the degree of smoothness imposed on the unknown velocity and transformation fields. In fact, the algorithm developed here tends to smooth over discontinuities both in the optical flow and transformation fields. One way to overcome this shortcoming is to use "line processes" (Marroquin [1984]). Simply put, the idea is to incorporate in the penalty function the cost of introducing a discontinuity in the optical flow or transformation fields instead of interpolating smoothly between two neighboring points when the gradient becomes large. A line process is a boolean variable; it takes on the value 1 where there is a discontinuity and 0, otherwise.

There is, however, a drawback associated with the introduction of line processes in the minimization scheme, namely, that the cost functional becomes non-convex. This generally calls for inventing sophisticated optimization methods that can be computationally exhaustive; for example, a simulated annealing scheme (Marroquin [1984]), or an algorithm based on neuronal network models (Hopfield & Tank [1985], Koch et al. [1986]).

Alternatively, we can employ the graduated non-convexity algorithm based on the concept of weak continuity constraints (Blake & Zisserman[1986]). Here, the process of minimizing a non-convex cost functional is replaced by minimizing a sequence of cost functions, the first of which is a convex approximation to the true cost functional and the last one is the true non-convex cost functional. Needless to say, we have yet to implement any of these schemes.

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